Atlas of Lie Groups and Representations
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Overview

1. Structure Theory
   (a) Root Data
   (b) Radical, Center, Derived data, dual
   (c) Inner classes (involutions)
   (d) Real Groups
      i. component group, Galois cohomology
      ii. center
      iii. Cartan subgroups and Weyl groups
      iv. Galois cohomology
   (e) Maximal Compact K
      i. K_0 as an algebraic group
      ii. disconnectedness
      iii. Parametrizing representations of K
   (f) K orbits on G/B
   (g) K orbits on G/P
2. The Langlands classification
   (a) standard and irreducible representations
   (b) Kazhdan-Lusztig-Vogan polynomials

   (c) decomposing standard modules/character theory
   (d) branching to K

3. Induction
   (a) parabolic subgroups
   (b) real parabolic induction
   (c) cohomological induction (Euler characteristic)

4. Hermitian forms
   (a) Hermitian representations
   (b) c-Hermitian form
   (c) signature of Hermitian forms in the equal rank case
   (d) *twisted KLV polynomials and the unequal rank case*

5. Character theory
   (a) KLV cells
   (b) translation functors
   (c) coherent continuation
   (d) Vogan duality

6. Advanced topics
   (a) Nilpotent orbits
   (b) *Associated varieties*
   (c) unipotent representations
   (d) *Lusztig-Bezrukavnikov conjecture on K-theory of the nilpotent cone*

Background

The lectures will assume some familiarity with:

1. Structure theory of reductive groups; for example Springer, *Linear Algebraic Groups* [4].

2. Basics of representations of real groups, $(\mathfrak{g}, K)$-modules; for example Chapter 1 of Vogan, *Representations of Real reductive groups* [5] or Chapters 1-3 of Knapp, *Overview* [3].

Atlas software

I'll be talking about the mathematics behind the atlas software, and the software itself. You may want to install the software on your laptop, although this isn't necessary. It is available for unix, Mac, and Windows. Go to www.liegroups.org/software. I can help installing it.

Alternatively, you can get an account on atlas.math.umd.edu and run the software there. See me for details.

The theory of real forms and strong real forms is covered in *Real forms and the Kac Classification* [2]. The basic reference for the mathematical background is *Algorithms for Representation Theory of Real groups* [1]. There are a number of other papers on the atlas web site www.liegroups.org/papers, especially in the *Read Me First* section.

Web Site

These notes and other material from the lectures are available at www.liegroups.org

References

[1] J. Adams and Fokko du Cloux, *Algorithms for representation theory of real reductive groups*, J. Inst. Math. Jussieu **8** (2009), no. 2, 209–259. MR2485793

[2] Jeffrey Adams, *Real forms and the kac classification.* www.liegroups.org/papers/realforms.pdf.

[3] Anthony W. Knapp, *Representation theory of semisimple groups*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, 2001. An overview based on examples, Reprint of the 1986 original. MR1880691 (2002k:22011)

[4] T. A. Springer, *Linear algebraic groups*, Second, Progress in Mathematics, vol. 9, Birkhäuser Boston Inc., Boston, MA, 1998. MR1642713 (99h:20075)

[5] David A. Vogan Jr., *Representations of real reductive Lie groups*, Progress in Mathematics, vol. 15, Birkhäuser Boston, Mass., 1981. MR632407 (83c:22022)